10. Division of Polynomials

- **Polynomial**

An algebraic expression in which the exponents of the variables are non-negative integers are called polynomials. For example, $3x^4 + 2x^3 + x + 9$, $3x^4$ etc are polynomials.

- **Constant polynomial:** A constant polynomial is of the form $p(x) = k$, where k is a real number. For example, –9, 10, 0 are constant polynomials.
- **Zero polynomial:** A constant polynomial '0' is called zero polynomial.

General form of a polynomial:

A polynomial of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where $a_0, a_1 \ldots a_r$ are constants and $a_n \neq 0$.

Here, $a_0, a_1 \ldots a_n$ are the respective coefficients of $x^0, x^1, x^2, \ldots x^n$ and n is the power of the variable x.

$a_n x^n, a_{n-1}x^{n-1} - a_0$ and $a_0 \neq 0$ are called the terms of $p(x)$.

- **Classification of polynomials on the basis of number of terms**
  - A polynomial having one term is called a monomial e.g. $3x$, $25t^3$ etc.
  - A polynomial having two terms is called a binomial e.g. $2t - 6$, $3x^4 + 2x$ etc.
  - A polynomial having three terms is called a trinomial. e.g. $3x^4 + 8x + 7$ etc.

- **Degree**

The degree of a polynomial is the highest exponent of the variable of the polynomial. For example, the degree of polynomial $3x^4 + 2x^3 + x + 9$ is 4.

The degree of a term of a polynomial is the value of the exponent of the term.

- **Classification of polynomial according to their degrees**

- A polynomial of degree one is called a linear polynomial e.g. $3x + 2$, $4x$, $x + 9$.
- A polynomial of degree two is called a quadratic polynomial. e.g. $x^2 + 9$, $3x^2 + 4x + 6$.
- A polynomial of degree three is called a cubic polynomial e.g. $10x^3 + 3$, $9x^3$.

Note: The degree of a non-zero constant polynomial is zero and the degree of a zero polynomial is not defined.

- **Division of a polynomial by a monomial using long division method**

Example:
Divide $x^4 - 2x^3 - 2x^2 + 7x - 15$ by $x - 2$.
Solution:

$$
\begin{array}{r l}
 & x^3 - 2x + 3 \\
\cline{2-2}
x - 2 \,\big) & x^4 - 2x^3 - 2x \;+ 7x - 15 \\
 & x^4 - 2x^3 \\
 & - \quad + \\
\hline
 & \quad - 2x \;+ 7x - 15 \\
 & \quad -2x^2 + 4x \\
 & \quad + \quad - \\
\hline
 & \qquad 3x - 15 \\
 & \qquad 3x - 6 \\
 & \qquad - \quad + \\
\hline
 & \qquad\quad -9
\end{array}
$$

Divison of polynomials by monomials also satisfy Division algorithm i.e., **Dividend = Divisor** × **Quotient + Remainder**

It can be easily verified that here $(x^4 - 2x^3 - 2x^2 + 7x - 15) = (x - 2)(x^3 - 2x + 3) + (-9)$.